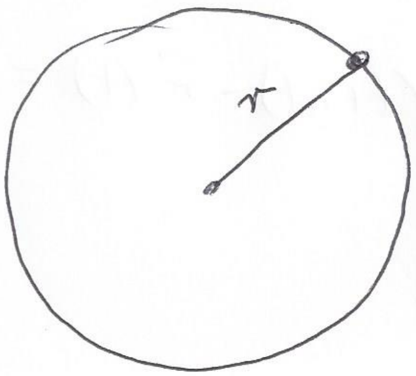
p1 <u>lecture 4</u>

- Vectors, unit vectors, circular motion
- Cross product.



circular motion at constant velocity

$$\frac{2\pi r}{T} = v \quad \text{velocity magnitude}$$

$$\left(\frac{2\pi}{T}\right) \cdot r = \omega \cdot r$$

$\omega = \frac{\theta}{t}$ = <u>angular velocity.</u> $\quad v = \frac{x}{t}$

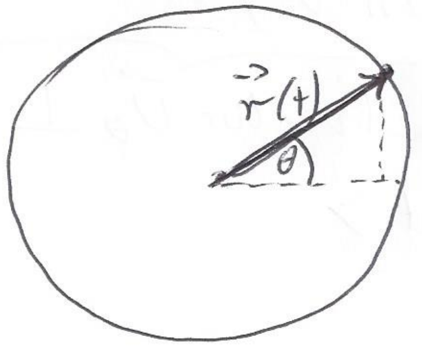
in radians/second!

$\omega(t) = \frac{d\theta}{dt}$ $\quad$ also called angular frequency. $\quad v(t) = \frac{dx}{dt}$

$$1\,rpm = \frac{2\pi}{60s} = \omega$$

$$\vec{A} = A \cdot \vec{U}_A$$

$$\vec{U}_{\vec{A}} = \frac{\vec{A}}{A}$$



$$\vec{r}(t) = \langle x(t), y(t) \rangle$$

$$x(t) = r \cdot \cos\theta = r \cdot \cos\omega t$$

$$y(t) = r \cdot \sin\theta = r \cdot \sin\omega t$$

$$\vec{r}(t) = r \underbrace{\langle \cos\omega t, \sin\omega t \rangle}_{\vec{U}_r(t)}$$

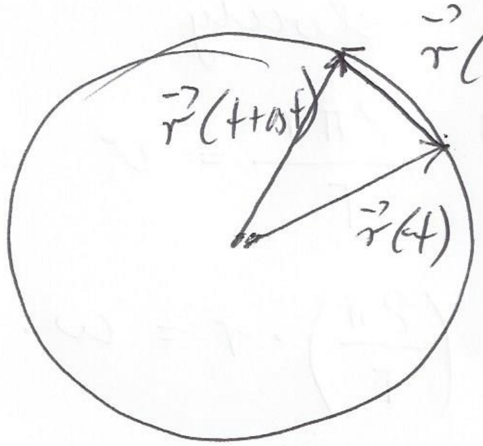
$$\vec{r}(t) = r \cdot \vec{U}_r(t)$$

$\vec{U}_r(t) = \langle \cos\omega t, \sin\omega t \rangle$

$\vec{r} = r \cdot \vec{U}_r$

$$\vec{v} = \frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$$

$\vec{r}(t+\Delta t) - \vec{r}(t) = \Delta \vec{r}$



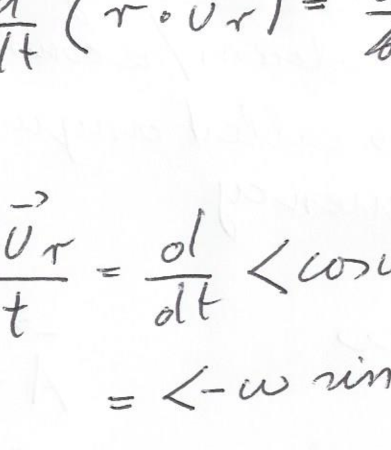
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r \cdot \vec{U}_r) = \underbrace{\cancel{\frac{dr}{dt}}}_{0} \vec{U}_r + r \cdot \frac{d\vec{U}_r}{dt}$$

$$\frac{d\vec{U}_r}{dt} = \frac{d}{dt} \langle \cos\omega t, \sin\omega t \rangle$$

$$= \langle -\omega \sin\omega t, \omega \cos\omega t \rangle$$

$$= \omega \underbrace{\langle -\sin\omega t, \cos\omega t \rangle}_{\text{unit vector } \vec{U}_\theta \perp \vec{U}_r}$$

$\vec{U}_\theta = \langle -\sin\omega t, \cos\omega t \rangle$



unit circle

$r = 1$

$\vec{r} = r\vec{U}_r \qquad \vec{v} = \omega r \cdot \vec{U}_\theta$

Find the acceleration:

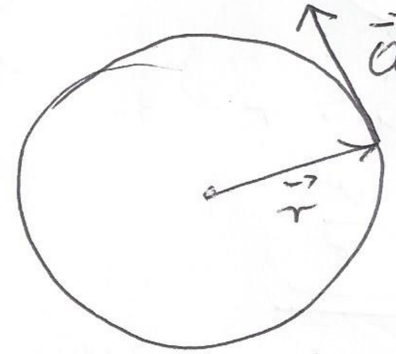
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\omega}{dt} \cdot r\vec{U}_\theta + \omega \cdot \cancel{\frac{dr}{dt}} \cdot \vec{U}_\theta + \omega r \cdot \frac{d\vec{U}_\theta}{dt}$$

$\frac{d\omega}{dt} = \alpha =$ angular acceleration; $\frac{dr}{dt} = 0$

~~tx~~

$\dot{\omega} \cdot r \cdot \vec{U}_\theta =$ tangential acceleration

$\vec{a}_\theta$; $a_\theta = \dot{\omega} \cdot r = \alpha \cdot r$



$\vec{r}$

If a_θ is constant $= \frac{dv_\theta}{dt}$; $v = a_\theta \cdot t + v_0$

displacement in the tangential direction $\cancel{s = \frac{dv}{d}} = v = \frac{ds}{dt}$

$$s = \frac{1}{2} a_\theta t^2 + v_0 \cdot t + s_0$$

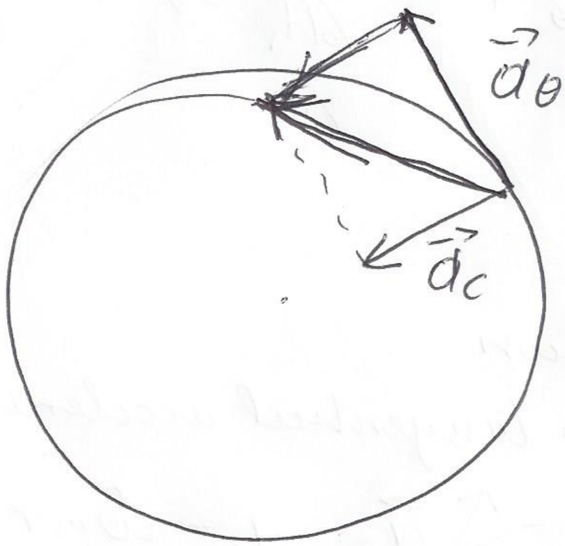
$$\frac{d\vec{U}_\theta}{dt} = \frac{d}{dt}\langle -\sin\omega t, \cos\omega t\rangle$$

$$= \langle -\omega\cos\omega t, -\omega\sin\omega t\rangle$$

$$= -\omega \underbrace{\langle \cos\omega t, \sin\omega t\rangle}_{\vec{U}_r}$$

$$\omega r \frac{d\vec{v}_\theta}{dt} = \omega r(-\omega \vec{v}_r)$$

$$= \underbrace{-\omega^2 r \cdot \vec{v}_r}_{\vec{a}_c \,=\, \text{centripetal acceleration}}$$



$$v = \omega \cdot r$$

$$\omega = \frac{v}{r}$$

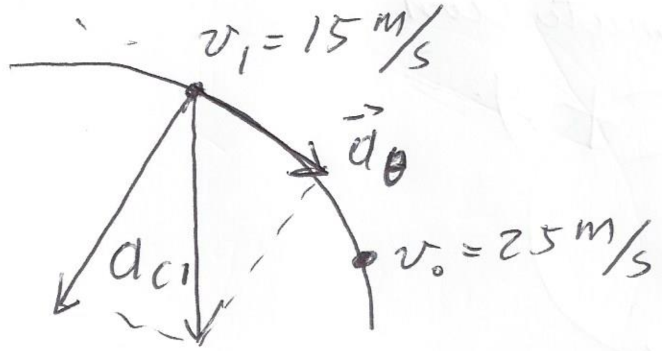
$$a_c = \omega^2 \cdot r$$

$$a_c = \frac{v^2}{r}$$

What is the velocity of a person on the equator as the earth turns: $v = \frac{2\pi r}{T} = \frac{2\pi \cdot 6.37 \cdot 10^6\,m}{24 \cdot 3600}$

$$= 463 \frac{m}{s}$$

A train enters a curve of radius 150m at a speed of 25 m/s and slows down to 15 m/s during the next 80 s.



$$a_\theta = \frac{v_1 - v_0}{t_1 - t_0} = \frac{15-25}{80} = \frac{-10}{80} = -0.125\ \frac{m}{s^2}$$

$$a_0 = \frac{v_0^2}{r} = \frac{25^2}{150} = 4.17\ \frac{m}{s^2} \qquad \vec{a}_0 = \vec{a}_c$$

$$a_{c1} = \frac{15^2}{150} = \frac{225}{150} = 1.5\ m/s^2$$

$$\vec{a}_1 = -0.125\ \frac{m}{s^2} \cdot \vec{U}_\theta + 1.5\ \frac{m}{s^2} \cdot \vec{U}_r$$

Cross product (vector product)

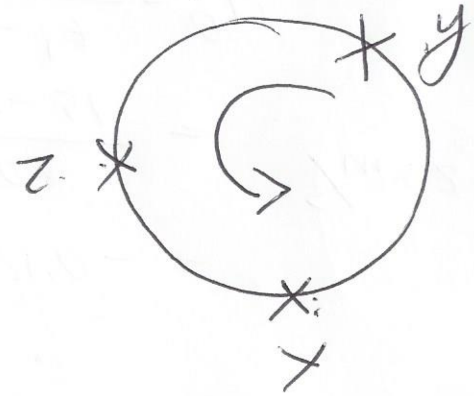
$\vec{A} \times \vec{B} = \vec{C}$ $\qquad$ $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$

$|\vec{C}| = A \cdot B \cdot \sin\theta$ $\qquad$ r.h.r

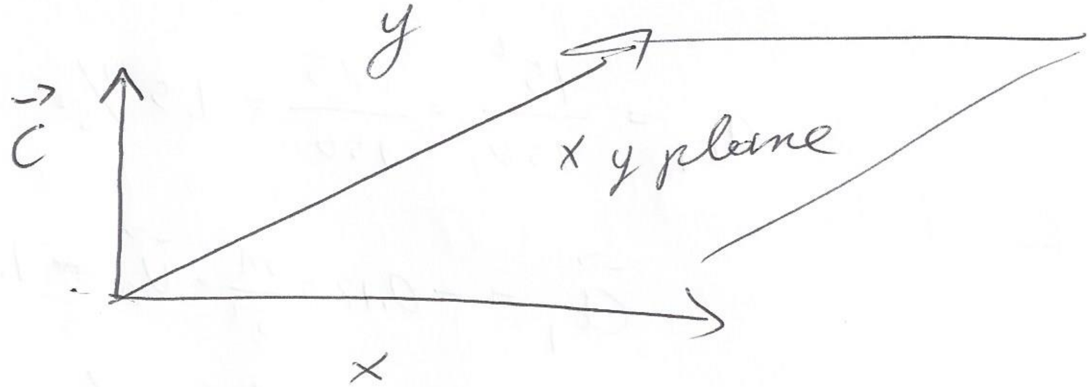
right hand rule

$$\vec{A} \times \vec{B} = \vec{C} = \langle C_x, C_y, C_z \rangle$$

$$\underbrace{C_x = A_y B_z}_{\text{circular permutation}} - A_z B_y$$



$$C_y = A_z B_x - A_x B_z \, ; \qquad \boxed{C_z = A_x B_y - A_y B_x}$$



$$\vec{A} = \langle 3, 5, 0 \rangle \qquad \vec{B} = \langle 1, 6, 0 \rangle$$

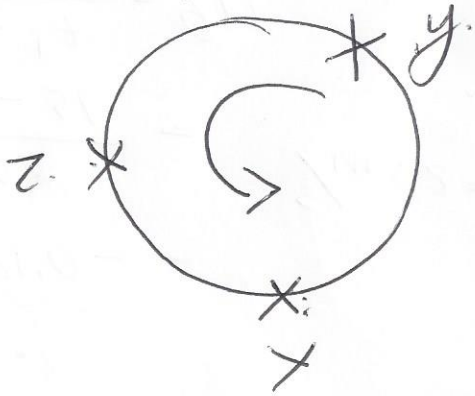
$$\vec{A} \times \vec{B} = C_z \cdot \vec{k}$$

$$C_z = 3 \cdot 6 - 5 \cdot 1 = \underline{\underline{13}}$$

If $\vec{A}$ and lie in the x-y plane the vector $\vec{C}$ has only a z-component.

$$\vec{A} \times \vec{B} = \vec{C} = \langle C_x, C_y, C_z \rangle$$

$$\underbrace{C_x = A_y B_z - A_z B_y}_{\text{circular permutation}}$$



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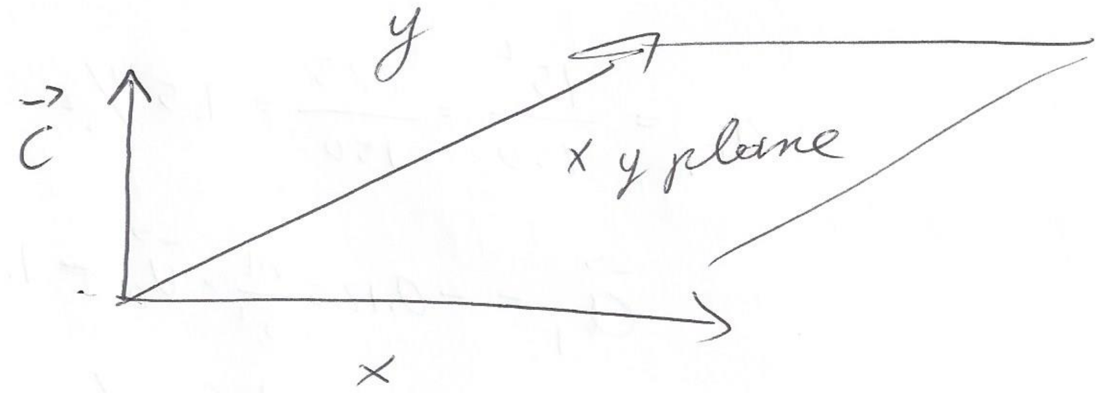

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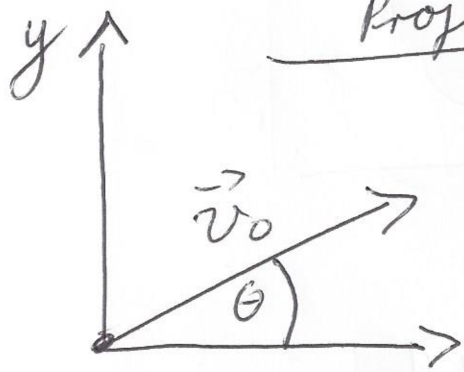
Mixed products:

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$abc = bac - cab$$

Projectile Motion



$$\boxed{x = v_{0x} \cdot t}$$

$$\frac{dx}{dt} = v_{0x}$$

$$a_x = 0$$

$$y = v_{0y} \cdot t - \frac{1}{2} g t^2$$

$$v_y = v_{0y} - g t$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

$$\vec{r} = \langle x, y \rangle$$

$$\vec{v} = \langle v_x, v_y \rangle$$

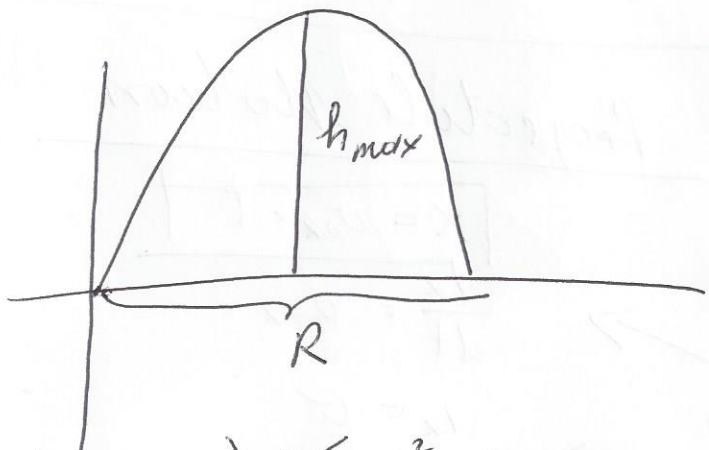
$$\vec{a} = \langle a_x, a_y \rangle$$

Find y as a function of x

$y = -\frac{1}{2} g t^2 + v_{oy} t$ $\qquad x = v_{ox} \cdot t$

$$t = \frac{x}{v_{ox}}$$

$$y = -\frac{1}{2} g \frac{x^2}{v_{ox}^2} + \frac{v_{oy} \cdot x}{v_{ox}}$$



<u>Find h_{max}:</u> $\quad \cancel{v_y^2} = v_{oy}^2 - 2gy$

$$y_{max} = \frac{v_{oy}^2}{2g} = \frac{v_o^2 \cdot \sin^2\theta}{2g}$$

<u>Find the maximum range!</u>

$y = 0 \quad v_{oy} \cancel{t} = \frac{1}{2} g t^{\cancel{2}}$

$$t = \frac{2 v_{oy}}{g} \ ; \quad x = v_{ox} \cdot t$$

$$x_{max} = R = range = \frac{v_{ox} \cdot 2 v_{oy}}{g}$$

$$= \frac{2 v_o^2 \cos\theta \cdot \sin\theta}{g}$$

$$\boxed{R = \frac{v_o^2}{g} \cdot \sin 2\theta}$$